Design of Maximum Thrust Nozzles for Nonequilibrium Chemically Reacting Flow

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Theme

THE first applications of optimization techniques to the design of maximum thrust nozzles were made by Guderley and Hantsch,[1] Rao,[2,3] and Guderley.[4] In those analyses, the problem was formulated to provide maximum thrust across a control surface through the nozzle exit for a design constraint of constant nozzle length. Since the optimization functional was expressed along the exit control surface, no dissipative effects in the flowfield could be accounted for, and design constraints not directly related to the exit control surface could not be included.

Guderley and Armitage[5] formulated the optimization problem including the entire flow region affecting the nozzle contour in order to apply design constraints directly related to the nozzle contour. The complexity of the formulation and the numerical solution procedure were greatly increased over the previous formulations. The Guderley-Armitage approach can be extended to dissipative flows since the entire flow region is considered in the formulation.

The present work presents the formulation and numerical method developed to determine maximum thrust nozzle contours for nonequilibrium chemically reacting flow. The optimization analysis is based on that developed by Hoffman.[6] The results of a parametric study are presented to demonstrate the capabilities of the method. A complete working computer program is available from the sponsoring agency.

Contents

The flowfield model for the optimization analysis is presented in Fig. 1. Changes in the nozzle wall contour only between points *I* and *F* in the supersonic region are considered. The fluid may be any multicomponent nonequilibrium chemically reacting gas. The thrust is expressed as the integral of the pressure forces along the wall *IF*, which is constrained to be a gas streamline. A general isoperimetric design constraint is applied along the wall. The governing gas dynamic equations are considered as constraints over the region *IKF*. The calculus of variations provides a set of design equations which suffice to construct a maximum thrust nozzle. Those equations and a numerical method for implementing their solution are presented in Ref. 7. The flowfield analysis was performed with the ICRPG kinetics program developed by Kliegel et al.[8]

Parametric studies were conducted to demonstrate the application of the method, and to investigate the magnitude of the potential gains. The following parameters were investigated: 1) kinetic rate uncertainty, 2) nozzle size, 3) stagnation pressure, 4) nozzle throat radii of curvature, and 5) off-design performance. Two different propellant combinations were studied: hydrogen-fluorine and hydrogen-air.

The nominal H_2-F_2 configuration described in Table 1 was considered. The reactions considered for this system are listed in Table 2. The reaction rates are those recommended by Cherry.[9]

Table 1 Nominal H_2-F_2 configuration

Chamber pressure	100 psia
Oxidizer/fuel ratio	7
H_2 enthalpy (ref. to 298.16°K)	−500 cal/g
F_2 enthalpy (ref. to 298.16°K)	−47 cal/g
Inlet cone half-angle	30°
Inlet contraction ratio	4
Throat radius	4.16 in.
Throat radius of curvature	8.32 in.
Nozzle length (from throat)	25 in.
Isoperimetric constraint	Constant length
Ambient pressure	0.001 psia

Table 2 Reactions for the H_2-F_2 system

Reaction	Reverse rate[a] A	X	B	Uncertainty
$F_2+M = F+F+M$	1.00×10^{18}	−1.5	0	±10
$HF+M = H+F+M$	7.50×10^{18}	−1.0	0	$\pm10^{1.5}$
$H_2+M = H+H+M$	7.50×10^{18}	−1.0	0	±10
$HF+F = F_2+H$	5.28×10^{12}	0.5	4.0	±25
$HF+H = H_2+F$	5.00×10^{12}	0	5.7	±50
$HF+HF = F_2+H_2$	1.75×10^{10}	0.5	3.97	$\pm10^3$

[a] Reverse rate $K_{rr} = AT^x e^{-B/RT}$ in cm, g-mole, °K, sec, kcal units.

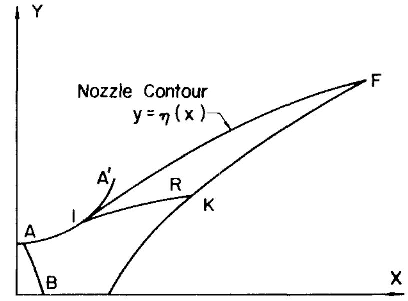

Fig. 1 Nozzle geometry and flowfield model.

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Maximum thrust nozzles were designed with the kinetic rates set at their upper and lower levels of uncertainty to illustrate the effects of rate uncertainty. Because of the appearance of small negative mass fractions for the trace species at the upper kinetic limit (a numerical problem in the ICRPG kinetics program, not the optimization method), no results were obtained for that case. The thrusts obtained for the different flow chemistry models are presented in Table 3. The results in Table 3 serve as a basis for evaluating the effects of nozzle size, stagnation pressure, throat radius of curvature, and off-design performance.

Table 3 Effect of reaction rates

Case	Chemistry model	Thrust, lbf
1	Equilibrium	9267
2	Upper kinetic limit	...
3	Nominal kinetic	9127
4	Lower kinetic limit	9036
5	Frozen	8767

The effect of nozzle size is illustrated in Table 4. Scaling the flow boundaries by a factor of two should result in a factor of four increase in thrust. The factor of four increases in cases 6 and 8 were achieved over their half-size counterparts within 0.02%. By contrast, case 7 produced nearly the equilibrium thrust, or over 400 lb more thrust than would be expected from a factor of four scaling.

Table 4 Effect of scaling

	Thrust, lbf	
Chemistry model	Nominal	Twice size
Equilibrium	(1) 9267	(6) 37062
Nominal kinetic	(3) 9127	(7) 36928
Frozen	(5) 8767	(8) 35061

Results for three different throat radii of curvature ρ_t, expressed in terms of the throat radius y_t, are presented in Table 5. In all cases, the small radius of curvature produced the maximum thrust, a result opposite to the intuitive feeling that a larger radius of curvature would yield a slower expansion rate in the throat and avoid freezing. This result is probably valid only for underexpanded nozzles.

Table 5 Effect of throat radius of curvature

	Thrust, lbf		
Chemistry	$\rho_t = 0.5y_t$	$\rho_t = 2y_t$	$\rho_t = 4y_t$
Equilibrium	(18) 9377	(1) 9267	(21) 9176
Nominal kinetic	(19) 9204	(3) 9127	(22) 9042
Frozen	(20) 8857	(5) 8767	(23) 8767

The effect of stagnation pressure is illustrated in Table 6. At $p = 100$ psia, the kinetic thrust is 98.49% of the equilibrium thrust, while at $P = 500$ psia, the kinetic thrust is 99.69% of the equilibrium thrust. That result shows that, as the stagnation pressure increases, kinetic flow approaches equilibrium flow.

Table 6 Effect of stagnation pressure

	Thrust, lbf	
Chemistry model	$P = 100$ psia	$P = 500$ psia
Equilibrium	(1) 9267	(15) 45831
Nominal kinetic	(3) 9127	(16) 45688
Frozen	(5) 8767	(17) 44106

Table 7 presents the off-design performance of the maximum thrust nozzles. Each nozzle design was analyzed with equilibrium, kinetic, and frozen flow chemistries. The major effect on thrust is seen to be the flow chemistry used in the analysis rather than the flow chemistry used in the design. These results suggest that, regardless of the kinetic rates used in designing a nozzle, the design will produce near the maximum thrust for whatever kinetic rates occur in the actual flow. In fact, the frozen flow design can be used with only a small degradation in actual kinetic performance.

Table 7 Off-design performance

	Thrust, lbf		
	Chemistry used in analysis		
Chemistry used in design	Equilibrium	Nominal kinetic	Frozen
Equilibrium	9267	9114	8758
Nominal kinetic	9248	9127	8766
Frozen	9243	9119	8767

Similar studies were conducted for the hydrogen-air system. The results were similar to those obtained for the hydrogen-fluorine system.

In conclusion, an optimization analysis and a computer program have been developed which permit the design of maximum thrust nozzles for the nonequilibrium chemically reacting flow of any general gas phase reacting mixture.

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